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On Predication, A Conceptualist View

Abstract: Predication, as the nexus between a subject and a predicate expression, is the basis of the unity of a speech act, including speech acts in the plural and speech acts that involve mass nouns. A speech act, of course, is an overt expression of a mental act, e.g., a judgment; and therefore the unity of a speech act such as an assertion is really the unity of the judgment that underlies that act. Such a mental act, and therefore the speech act as well, has a unity based on how the referential and predicable roles of the subject and predicate expressions combine and function together respectively. What we propose here is to explain this unity of predication in terms of a conceptualist theory of logical forms that we claim underlies at least some important aspects of thought and natural language. Our conceptualist logic also contains an account of the medieval identity (two-name) theory of the copula, as well as an account of plural and mass noun reference and predication, the truth conditions of which are based on a logic of plurals and mass nouns.

Keywords: predication, reference, pluralities, mass nouns, Conceptualism, cognitive capacities, copula, unsaturatedness, Medieval Logic,

0. Introduction

A speech act such as an assertion is not just a jumble of words, nor is the judgment that underlies it a stream of mental flotsam. An assertion and the judgment underlying it have a unity, something that accounts for the coherence of the judgment and the assertion it underlies. What is the basis of that unity or coherence? How can we account for it? It cannot be the rules of grammar that tells us how the words and phrases are to be correctly put together so as to express a judgment or a thought. Most people do not know the rules of grammar even after they have become competent in speaking and expressing their thoughts. So just what does hold the parts of an assertion or of a thought together?

As a speech act, an assertion is an overt expression of a mental act, i.e., of a judgment, and therefore the unity or coherence of such a speech act is really the unity or coherence of the mental act that underlies it. This unity, we claim, is based on how the referential and predicable concepts of the subject and predicate expressions of such an act combine and function together. Such a combination and mutual functioning of referential and predicable concepts is what we mean here by predication. What we propose is to explain the unity of predication in

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terms of a conceptualist theory of logical forms that represent referential and predicable concepts and that we believe explains at least some important aspects of thought and natural language. Our conceptualist logic also contains an account of plural and mass noun predication, i.e., of plural and mass noun reference and predication, the truth conditions of which are based on a logic of plurals and mass nouns that we have developed elsewhere.[1]

Concepts in this conceptualist theory are the intersubjective cognitive capacities humans acquire in learning a language and that subsequently come to underlie our use of language. Predicable concepts, in particular, are the cognitive capacities underlying our rule-following use of predicate expressions, and referential concepts are the cognitive capacities underlying our rule-following use of referential expressions. As cognitive capacities, concepts are the basis of a kind of pragmatic knowledge, or more specifically a knowing how to do things with referential and predicable expressions in various contexts of use of language and thought. Having such concepts is not a form of propositional knowledge, i.e., knowledge that certain rules of language are correct, even though having such concepts underlies the rule-following behavior those rules might describe. As intersubjectively realizable cognitive capacities that might be exercised at the same time by different persons and at different times by the same person, referential and predicable concepts are in an appropriate sense objective cognitive universals.

According to Kant, what unifies a mental act is a "synthetic unity of apperception of the thinking subject".[2] Such an apperception, Kant claimed, not only unifies each judgment, but it also determines the categorial structure of the different possible judgments that can be made. In fact, according to Kant, what categories there are and how they fit together is determined by a "transcendental deduction", and the categories so deduced form the basis of a so-called *transcendental logic*. But just how the synthetic unity of apperception unifies a judgment is not clear. Apparently, it is assumed that the unity is somehow determined by the mind of the thinker. The question then is just how does the mind do this? Is there a way we can logically represent this unity? We believe that the theory that we propose here for our conceptualist logic might possibly be taken as providing an answer to this last question, but in no sense do we claim that our logic is based on either a transcendental deduction or a synthetic unity of apperception.

Some philosophers claim that predication can be explained only in terms of a relation or "tie" of exemplification. Exemplification is said to be what holds

1 See Cocchiarella (2002) and (2009).

2 Kant (1965, §16), "The Original Synthetic Unity of Apperception".

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together (and hence unifies) what the subject of a sentence denotes with what the predicate stands for, the result being a state of affairs or a proposition. For the sentence 'Socrates is wise', for example, exemplification is what relates or ties Socrates with wisdom. Wisdom of course is an abstract entity that does not exist in the world the way Socrates does (or did). That is, wisdom exists – or rather has *being* if *existence* applies only to things in the physical world – in some abstract platonic realm. Exemplification, accordingly, is assumed to be a relation or "tie" between something concrete or physical and something abstract. Aside from the question of how there could be a such a relation or "tie" between objects of different realms or modes of being, this sort of answer does not explain how the referential and predicable concepts that are exercised in a judgment are unified in a speech or mental act. After all, a mental act, such as a judgment, does not have either a concrete object such as Socrates or an abstract object such as wisdom as a constituent. A relation of exemplification, in other words, is not what explains the unity of a speech or mental act.

Some other philosophers, including especially Gottlob Frege, would object to a relation of exemplification because it replaces the predicate phrase, e.g., 'is wise', by its nominalized form, 'wisdom', an objection with which we agree. The predicate phrase in 'Socrates is wise' does not function as a name, in other words, and that is because what the predicate stands for is not an object (or thing) that can be named. According to Frege, what a predicate stands for is an abstract unsaturated function from objects to truth values, which Frege also called a concept, but not in the sense of a cognitive capacity. Instead of exemplification, what the sentence 'Socrates is wise' says is that Socrates falls under the abstract concept that the predicate stands for, i.e., the abstract function that 'is wise' stands for assigns truth to Socrates. This answer is also unsatisfactory, because as an abstract entity a Fregean concept is not a mind dependent entity, and therefore it has nothing to do with the unity of a mental act.

Finally, whether predication is explained in terms of exemplification or an abstract Fregean function from objects to truth values, many philosophers view the judgment that 'Socrates is wise' as expressing a belief, which is then assumed to be a relation between a person's mind and a proposition (or state of affairs) having Socrates and the property of being wise as constituents. Be that as it may, such a view still does not account for the unity of the judgment as a speech or mental act, and therefore we will not be concerned with such a view here, whether positively or negatively.

These kinds of answers, and others like them, are semantical accounts of the truth conditions of a speech act, and in particular a speech act of rather simple

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form where the subject is either a proper name or a definite description. None really explains (or purports to explain) what accounts for the unity of a speech or mental act. What these sorts of theories purport to explain are the truth conditions of simple subject-predicate sentences, from which, supposedly, the truth conditions of other kinds of sentences are then determined as in a model-theoretic semantics. We do not dispute the importance of such an account of truth conditions, but whatever the significance of this approach it does not explain the problem of the unity of a speech or mental act. We do assume, as far as truth conditions are concerned, that the logical forms of our conceptualist theory provide a perspicuous guide to what logically follows from what, and in that regard the theory provides all of the semantics that we need for our present purpose. In other words, we assume that the logical forms of our underlying logic carry their semantics on their sleeves.

1. General Reference and the Medieval *Suppositio* Doctrine

Subject expressions of natural language, and of English in particular, consist not only of proper names or definite descriptions, but also of quantifier phrases such as 'Every republican who voted for the bill' and 'Some democrat who did not vote for the bill'. As subject expressions of speech acts, these quantifier phrases, are used to express different forms of general reference, and the unity of a speech act based on the use of such an expression must be accounted for no less so than those having a proper name or definite description as a referring subject expression. Our theory, in other words, must account for general reference as well as singular reference. Here again one must not confuse accounting for the truth conditions (as in standard model theory) of these general statements or judgments in terms of their instances with explaining their unity as speech acts or judgments. That in effect amounts to changing the subject.

Historically these kinds of speech and mental acts were taken seriously by medieval logicians and linguists, especially in the 14th c. They were explained primarily by the medieval *suppositio* (supposition) theory of William of Ockham, John Buridan, Walter Burley, Gregory of Rimini, and a number of others. Today this theory is generally called *terminist logic* because of its focus on the categorematic expressions of Latin that were called *terms*. As in our conceptualist theory, reference in terminist logic was a pragmatic notion, i.e., it was intended to apply to our speech and mental acts in different contexts of use.

One important part of *suppositio* theory is the doctrine of the modes of supposition. This doctrine, despite the reference to "modes" of *supposition*, is not a theory about different "ways of referring". Rather, the "modes" are just different

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types and subtypes of *personal supposition*, which is reference to things. We have explained elsewhere how these different subtypes of personal supposition, and Ockham's *suppositio* theory in general, can be reconstructed in terms of our conceptualist theory of reference, and the details of that analysis will not be repeated here.[3] Instead, we briefly consider the kind of language that was implicit in terminist logic, and then in a later section discuss the terminist identity (or two-name) theory of the copula, which will be useful in our account of predication.

Implicit in terminist logic was the view that there is a language of thought that today is sometimes called *Mental*, a language that was made up of both categorematic and syncategorematic concepts.[4] This mental language was assumed to be a "natural language" common to all humans and somehow established by nature. What makes Mental natural is that its categorematic concepts (mental terms) get their signification (reference) by nature and not by convention. The assumption was that there is a "natural likeness" between concepts and the things they signify (refer to), a likeness that was caused by the things signified.

Implicit in this theory was the idea that concepts are like images that resemble things, a view that is quite different from the kind of cognitive capacities that we take them to be in our form of conceptualism. For a variety of reasons that we will not go into here, this "imagist" theory is an inadequate account of concepts. One reason is that many of the concepts that are cognitive capacities in our form of conceptualism, such as the concepts we have in mathematics, and the various concepts we have about the microphysical world, can in no sense be said to have a likeness to things in nature.[5] Otherwise than this sort of difference in the nature of concepts much of what the medieval philosophers and linguists have to say about Mental can be explained in terms of the logical framework of our form of conceptualism.

Mental is a tensed and modal language containing among its syncategorematic concepts certain operators that correspond to the tenses and modal modifications of verbs, or what the medieval logicians called ampliation. Our conceptualist logic includes these features and represents them as formula operators, but for convenience we will not deal with the details of tense and modal logic in this paper – except briefly later in order to deal with a specific problem regarding predication based on relations. Also, because of ampliation mental terms were taken to signify (refer) in a wider sense than just to (presently) existing things. In

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3 See Cocchiarella (2001). See also Spade (2007).
4 See, e.g., Geach (1980b), Normore (1990), and Trentman (1970). See also Scott (1966b).
5 For more on this difference between *Mental* and our form of conceptualism see Cocchiarella (2001, §2).

other words, Mental was ontologically committed not just to present but also to past and future objects, and even possible objects. The possible objects that were signified (in the wider sense), however, seemed to be only those that are possible in nature.[6] In our form of conceptualism we also allow for the kind of possibilia we believe to be implicit in natural language, but we distinguish the many concepts that entail existence such as *dog*, *house*, *man*, etc., and even *dragon*, from those that do not. In other words, something cannot be a dog, house, man, or a dragon unless it exists. But then we also distinguish fictional and mythological contexts from ordinary contexts of use, so that although no dragons exist in reality, some dragons exist in fiction.[7]

According to Geach, the "grammar of Mental turns out to be remarkably like Latin grammar", and indeed Ockham did seem to carry over some of the features of Latin into Mental.[8] But according to J. Trentman, "Ockham's real criterion (…) for admitting grammatical distinctions into Mental amounted to asking whether the distinctions in question would be necessary in an ideal language – ideal for a complete, true description of the world".[9] Similarly, according to many medieval specialists, the proper comparison is not of Mental with Latin but of Mental with the kind of "ideal languages" that logicians and philosophers have constructed in the twentieth century, i.e., with the kind of logical system such as our conceptualist theory of logical form.

What is needed for an adequate interpretation of terminist logic is a representation of Mental as a logistic system, and in particular one based on a view of logic as language as opposed to logic as calculus. We take the logic of our conceptualist theory of predication to be just that sort of system. We turn now to the first part of our version of such a logic.

2. General Reference in the Logic of Names

The initial form of our theory of reference is called a logic of names, where by names we mean not just proper names but also common nouns, whether simple or complex, and even verbal nouns complex or simple. Verbal nouns are used to refer to events, as when we speak of a kissing, or a jumping, etc. For convenience

6 See Normore (1985: 191). We note that a natural possibility and necessity seems to be what Burley had in mind.

7 See Cocchiarella (2013) for more on the distinction between existence-entailing concepts and concepts that do not entail existence.

8 See Geach (1980b).

9 See Trentman (1970: 589).

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we will not deal with verbal nouns in this paper. In regard to names otherwise than verbal nouns, we note that medieval terminist logicians held, as did Leśniewski in his logic of names (which he also called ontology), that the category of names includes common names, i.e., common nouns, as well as proper names. This is different from most views today where common names are taken as predicates and proper names, along with definite descriptions, are taken as singular terms, which is an entirely different category from predicates. Peter Geach, incidentally, changed his mind on this matter and came to agree with Leśniewski's and our view of names. Geach, in other words, "came to accept the view of Leśniewski and other Polish logicians that there is no distinct category of proper names (….) in syntax there is only the category of names".[10]

We take it that a basic form of judgment is expressed by an assertion that consists of a noun phrase and a verb phrase, and that the noun phrase has a referential role regardless whether or not it is a proper name, a definite description, or a quantifier phrase. A definite description is viewed in our theory as a quantifier phrase on a par with a universal or existential quantifier phrase.[11]

But because our first-order logic is free of existential presuppositions, a definite description may fail to refer to anything, as might the use of a proper name as well.

A quantifier phrase in our logic of names is made up of two parts, the first being a determiner such as 'every', 'some', the indefinite article 'a', and the definite article 'the' – and others as well, such as 'most', 'few', etc., which we will not deal with here. The second part of a quantifier phrase is a name in our present sense, which may be a proper name or a common name, i.e., a common noun, which could be a mass noun, a count noun, or a gerund in its role as a verbal noun (and which, as noted, we will not deal with here). A count noun can be simple, such as, e.g., 'politician', or complex, such as 'politician who is conservative', where the complexity is the result of affixing a qualifying relative clause, such as 'who (or that, or which) is conservative', to the head noun. Similarly, a mass noun can be simple, such as, e.g., 'water', or complex as with 'water that is polluted'.

Our basic logic, which we call a logic of names, is an extension of free first-order predicate logic with identity, i.e., first-order predicate logic free of existential presuppositions. The extension consists of adding a new syntactic category, which we call the category of names (in our present sense). The name variables of this category may be bound by either the universal or existential quantifiers. Name variables and constants are the simple names in this category. Complex names

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10 Geach (1980b: 15).

11 Evans held a similar view of definite descriptions in (1982: 57).

are formed by means of a relative-clause operator. Leśniewski's logic of names is different from ours, but, as we have shown in Cocchiarella (2001), Leśniewski's logic of names is reducible to our logic of names, i.e., Leśniewski's logic can be translated into our logic so that every theorem of his system is a theorem of ours.[12]

Our logic of names contains absolute (unrestricted) as well as restricted, relative quantifier phrases, i.e., relative quantifier phrases such as $(\forall xA)$ and $(\exists xA)$, read as 'every A' and 'some *A*', respectively, where *A* is a name, common or proper, and complex or simple.[13] We will use the standard quantifier forms $(\forall x)$ and $(\exists y)$ for the absolute quantifier phrases. We use *x, y, z,* etc., with or without numerical subscripts, as first-order variables and *A, B, C,* with or without numerical subscripts, as name variables. Complex names are formed by adjoining defining or restricting relative clauses to names. For our relative-clause operator we use '/', as in $A/\varphi x$, to represent the adjunction of a formula φx to the name *A* (which may itself be complex). We read $A/\varphi x$ as '*A* that is φx'. Thus, e.g., the quantifier phrase representing reference to *a brown dog* can be symbolized as $(\exists x Dog/Brown(x))$. We assume that an attributive adjective, such as 'brown' in 'brown dog', is equivalent to its occurrence as a predicate in a relative clause, as in 'dog that is brown'. Attributive adjectives such as 'alleged' in 'alleged thief' are really operators, as in 'person who is alleged to be a thief', where 'alleged' would be symbolized as the operator 'it is alleged that'.[14] A relative clause might itself be complex, of course, which is then represented by an iteration of the operator /. The complex quantifier phrase 'a brown dog that is vicious', for example, can then be represented as $(\exists x Dog/Brown(x)/Vicious(x))$.

A proper name, such as 'Gina', for example, can occur in our logic as part of a quantifier phrase, as in $(\exists x Gina)F(x)$, which indicates that the name 'Gina' is being used with existential presupposition. In our logic of plurals (and mass nouns), which we describe later as an extension of our logic of names, all names, proper or common, can be transformed into "terms", i.e., arguments of predicates, and one result of such a transformation is that the formula $(\exists x Gina)F(x)$ turns out to be equivalent

12 See Cocchiarella (2001) for a proof of this reduction.

13 Absolute quantifiers will range over not just every object or thing, but every plurality as well, or what later we will call classes as many. As we note later, this will include what mass nouns denote as well.

14 Some attributive adjectives such as 'big' and 'small', as in 'big mouse' and 'small elephant' have an analysis more involved than as simple predicate adjectives; but we will not deal with those adjectives here.

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to the more standard free-logic expression.[15] Thus whereas $(\exists x Gina)F(x)$ represents the referential role of the name 'Gina' in a speech act, the formula $(\exists x)[x = Gina \wedge F(x)]$ represents the truth conditions and logical implications of that speech act.

As already noted, definite descriptions are also quantifier phrases in this logic no less so than indefinite descriptions. As quantifier phrases, both definite and indefinite descriptions can be used as referential expressions, though of course they differ logically (and therefore semantically) in their referential roles. Definite descriptions, as indicated, can also be used with or without existential presupposition. We will use $\exists_1$ for the definite description operator when it is used with a presupposition and $\forall_1$ when without. Our analysis of a use of $\exists_1$ agrees in essentials with Bertrand Russell's contextual analysis of definite descriptions. We symbolize 'The *A* is *F*' as $(\exists_1 xA)F(x)$ when 'The *A*' is used with existential presupposition, and as $(\forall_1 xA)F(x)$ when used without.[16] Thus, e.g., an assertion of 'The black dog is vicious', where the definite description is being used with existential presupposition is symbolized as:

$$(\exists_1 xDog/Black(x))\,Vicious(x),$$

which represents the form of the speech act, whereas

$$(\exists xDog/Black(x))[(\forall yDog/Black(y))(y = x) \wedge Vicious(x)]$$

gives a more perspicuous representation of its truth conditions. Applying meaning postulates about how relative clauses and relative quantifiers are to be expanded, this last can be further expanded as follows:[17]

$$(\exists x)[(\exists yDog)(x = y) \wedge Black(x) \wedge (\forall z)((\exists yDog)(z = y) \wedge Black(z) \rightarrow z = x) \wedge Vicious(x)].$$

15 This use of "term" should be distinguished from what the medieval logicians meant by a term.

16 The truth conditions of these formulas are given more perspicuously as,

$$(\exists_1 xA)F(x) \leftrightarrow (\exists xA)[(\forall yA)(y = x) \wedge F(x)],$$

for situations where the definite description is used with existential presupposition, and as

$$(\forall_1 xA)F(x) \leftrightarrow (\forall xA)[(\forall yA)(y = x) \rightarrow F(x)]$$

where used without.

17 The meaning postulates in question here are as follows:

$$(\forall xA/F(x))G(x) \leftrightarrow (\forall x)[(\exists yA)(x = y) \wedge F(x) \rightarrow G(x)]$$

and

$$(\exists xA/F(x))G(x) \leftrightarrow (\exists x)[(\exists yA)(x = y) \wedge F(x) \wedge G(x)].$$

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This formula gives even more details about the truth conditions in question. But these are matters about the background logic and truth conditions of our speech and mental acts, as opposed to the logical forms of our speech or mental acts themselves as described in terms of the referential and predicable concepts being exercised. In other words, the first of the above formulas represents the logical form of the speech act in question, namely the assertion that the black dog is vicious, whereas the last provides a more perspicuous representation of its logical form with respect to its deductive implications, and therefore its truth conditions as well. This distinction between the logical forms that represent the mental or speech-act level and the logical forms that represent the deductive truth-conditional level is a fundamental feature of our conceptual logic, and it is important that it be kept in mind in understanding how this theory works.

Finally, we should note that *Dog* is not a predicate in our logic but a common name, and $Dog(x)$ is neither well-formed nor the proper way to symbolize the statement that x is a dog. Instead we use $(\exists y Dog)(x = y)$ to say that x is *a* dog. The indefinite article is both retained and represented in our account, which is as it should be in a representation of our speech acts.

3. On the Unsaturated Nature of Concepts

Now what quantifier phrases stand for as noun phrases in the subject positions of our speech acts are referential concepts, and of course what predicate expressions stand for in such acts are predicable concepts. What unifies the speech act, and the underlying mental act, is the joint exercise of a referential and a predicable concept. As intersubjectively realizable cognitive capacities, these concepts have an unsaturated nature each complementary to the other, so that when exercised each saturates the other in a kind of mental chemistry resulting in a speech or mental act. The mutual saturation corresponds to what Gottlob Frege described as a first-level concept *falling within* a second-level concept, which is appropriately represented by the adjunction of a quantifier phrase with a predicate expression. We note that even though *falling within* is not same as *falling under*, i.e., where an object is said to fall under a first-level concept, nevertheless, according to Frege, both an object falling under a first-level concept, and a first-level concept falling within a second-level concept result in a truth value. That, however, is not how we understand the unsaturated nature of referential and predicable concepts.

In particular, as cognitive capacities, a referential and predicable concept, when exercised together in a speech or mental act, result in an event, not a truth value. The event may be just a mental event, e.g., a judgment, if it is not overtly expressed, or a speech act event as well if it is overtly expressed. The exercise of a referential

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concept informs the event with a referential, directed nature (or what Brentano and Husserl called a *presentation*[18]), and the exercise of a predicable concept informs the event with a predicable nature.

The unsaturated nature of referential and predicable concepts as cognitive capacities is radically different from the unsaturated nature of Frege's concepts. Frege's notion of a concept is that of an abstract function from objects to truth values, which means that it is nothing at all like a cognitive capacity. The unsaturated nature of concepts as cognitive capacities, on the other hand, is analogous to the nature of dispositional properties that real, non-abstract things might have, except that dispositions have a "were-would" nature, whereas cognitive capacities have a "were-could" nature. That is, if something has a dispositional property (such as solubility in water), then if it *were* in a certain context or placed under certain circumstances, then that thing *would* have a related but different property (such as being dissolved in water), whereas if a speaker *were* in an appropriate linguistic or mental context to apply a given concept, then that speaker *could*, but might not, exercise that concept in that context. This quasi-dispositional nature of concepts as cognitive capacities explains how concepts, unlike the momentary image-concepts of the medieval *suppositio* theory, can continue to exist as capacities even when they are not being exercised – just as dispositional properties somehow exist in nature even when they are not being manifested. It also explains how the same concept as an intersubjectively realizable rule-following cognitive capacity can be possessed, and exercised, by different people at the same time, as well as by the same person at different times. The unsaturated nature of concepts as cognitive capacities is a natural property that concepts have, and therefore, unlike Frege's notion, it is not part of an abstract ontology. Also, as the cognitive capacities underlying our rule-following abilities in the use of referential and predicable expressions, concepts have a certain kind of functionality different from Frege's abstract notion, specifically in how they are exercised and function in speech and thought. This kind of functionality explains their complementarity, i.e., how they mutually saturate each other in thought and speech.[19]

The exercise of a referential concept informs a speech or mental act (an event) with a referential nature, just as the predicable concept jointly exercised with

18 For a formal representation of Brentano's notion of a presentation in terms of the present logic of names see Cocchiarella (2013).

19 We do not claim that the unsaturated nature of concepts as cognitive capacities are irreducible to neurophysiological properties of the brain. As with the dispositional properties of physical objects, the issue of reducibility to occurrent states or properties remains controversial.

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that referential concept informs that act with a predicable nature. Thus, every affirmative assertion that is syntactically analyzable in terms of a subject phrase and a predicate phrase (regardless of the complexity of either) is semantically analyzable in terms of an overt exercise of a referential concept with a predicable concept – and the assertion itself, as an event, is just the mutual saturation of their complementary structures in that speech act.

4. Nominalization and Relational Predication

We now extend our free first-order logic of names to a second-order predicate logic in which predicate expressions can be nominalized and occur as arguments of predicates, including of themselves. Given certain constraints that we will not go into here, Russell's paradox is not forthcoming, and the result is provably equivalent to the theory of simple types.[20] Predicate variables and quantifiers binding such are of course part of the logic. We will use the same symbol for the nominalized form of a predicate variable or constant. But when a predicate variable or constant occurs in its predicative role (along with one or more arguments), then it must have a pair of accompanying parentheses, as in $F(x)$ or $G(z, y)$, whereas the parentheses are deleted when the variable or constant occurs as a term or argument of a predicate, as, e.g., in $G(z, F)$, where G occurs as a predicate with its parentheses and F occurs as a term without parentheses.[21] We use the variable-binding λ-operator to represent complex predicates, as in the Russell predicate, $[\lambda x(\exists F)(F = x \wedge \neg F(x))]$, which is read as 'to be a concept that does not fall under itself', and which *qua* predicate expression stands for a predicable concept, i.e., a value of the bound (one-place) predicate variables, but which when nominalized does not denote a value of the bound first-order variables of our free first-order logic, because otherwise the paradox would be generated. In reasoning and trying to derive a contradiction on the basis of the Russell predicate we must know how to use the predicate, which means that the predicable concept for which it stands must exist as a cognitive capacity, otherwise we would not be able to reason with it. It does not mean, on the other hand, that the intensional content of the concept can be "objectified", i.e., exist as a value of the bound first-order variables.

What nominalized predicate expressions denote (when they in fact denote) are the intensions (or intensional contents) of the concepts (cognitive capacities) they stand for in their role as predicates. These intensions can be informally identified

20 See chapters 5 and 6 of Cocchiarella (1986), or §4.5 of Cocchiarella of (2007).

21 Our use of 'term' here should not be confused with what medieval logicians called terms.

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with the functions from possible contexts of use of the predicates to the extensions of the concepts the predicates stand for in those contexts. They cannot of course be the concepts themselves, because as unsaturated cognitive concepts, the latter cannot be taken as values of the bound first-order variables. When a nominalized predicate occurs in a formula, we say that the concept the predicate stands for as a cognitive capacity has been *deactivated* (with respect to the nominalized occurrence of the predicate). It is deactivated of course because it is no longer functioning in that occurrence as a predicate.

One important use of nominalized predicates is for their occurrence as direct objects of intensional verbs, as in 'Jim wants to be president', which we can symbolize as:

$$(\exists x Jim) Wants(x, [\lambda y President(y)]),$$

where the nominalized predicate $[\lambda y President(y)]$ is read as the infinitive phrase 'to be president'. We should note that the above predication, as well as all of the predications based on a relation discussed in this chapter, could (and perhaps should) be formulated in terms of a (complex) monadic predication as a λ-abstract. That would make it clear that what appears to be a relational predication is really a complex form of monadic predication, i.e., where predication is explained in terms of the mutual saturation of a referential concept and a monadic predicable concept. Thus, e.g., in the case now being discussed, we could (and perhaps should) symbolize the above sentence as:

$$(\exists x Jim)[\lambda x Wants(x, [\lambda y President(y)])](x).$$

We avoid doing so here mainly for simplicity and convenience of expression. Our point remains that predication is the mutual saturation of a referential and a (monadic) predicable concept even when the predicate expression is a complex based on a relation. We should note here, incidentally, that as a cognitive capacity a predicable concept, and similarly a referential concept, is not assumed to have a complex structure corresponding to the predicate, or referential expression, whose use it underlies – as might well be the case in a realist theory of predication about properties and properties of properties.[22]

In the above example we are not using 'is president' as a predicate. The predicate has been nominalized, indicating that the concept it stands for has been deactivated. Our view that a nominalized predicate occurring as direct

22 Properties of properties are what quantifier phrases stand for in a realist theory of predication.

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(or indirect) object of a relational verb in a speech act denotes the intension of that predicate is similar to Richard Montague's view that such an occurrence is to be taken as denoting the sense of the predicate.[23] Montague gave this sort of analysis for the direct objects of all relations, whether or not those relations were intensional or extensional. That was an important insight especially with respect to how we should analyze direct (or indirect) objects that contain quantifier phrases. We will adopt the same strategy here for speech or mental acts, except that, instead of denoting the sense of a predicate occurring as the direct (or indirect) object, we take the occurrence of the predicate to be deactivated, which means that it is nominalized and denotes its intension instead.[24]

Thus, consider Montague's example of 'Jim wants to find a unicorn', which is symbolized as:

$$(\exists x Jim) Wants(x, [\lambda y(\exists z Unicorn) Finds(y, z)]).$$

Here the nominalized (complex) predicate $[\lambda y(\exists z Unicorn) Finds(y, z)]$ is read as 'to be a *y* such that *y* finds a unicorn', and here again on our view the predicate 'finds a unicorn' has been deactivated and is not being used to predicate finding a unicorn of anyone. The phrase 'a unicorn' occurs in this context as a quantifier phrase, but because it occurs within a deactivated predicate it is then understood as being itself deactivated, and hence as representing a deactivated referential concept. In other words, there is no reference to a unicorn in this example.

Now we do sometimes represent a quantifier phrase as being directly and not indirectly deactivated and hence occurring as a term of a complex predicate. The example Montague considered for his sense-denotation type theory was 'John seeks a unicorn', where the quantifier phrase 'a unicorn' is the direct object of 'seeks'. Instead of dealing with the sense of the quantifier phrase 'a unicorn' as in Montague's theory, we nominalize or deactivate it by first correlating the phrase with a predicate as follows:

$$[\exists x Unicorn] =_{df} [\lambda y(\exists G)(y = G \wedge (\exists x Unicorn) G(x))],$$

23 See Montague (1970).

24 The difference between senses and intensions can be important, as in Montague's theory, or not, as in our theory, where much depends on how fine-grained an analysis is given to intensions. In our theory much depends on how we understand a context of use of language.

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and then we nominalize or deactivate the predicate.[25] Here the predicate in question can be read as 'to be a concept under which a unicorn falls'. The intension of this predicate is what we take as the intension of the quantifier phrase 'a unicorn'.

The sentence 'John seeks a unicorn' can then be symbolized as follows:

$$(\exists yJohn)Seeks(y, [\exists xUnicorn]).$$

A similar analysis applies to 'John fi nds a unicorn' as well:

$$(\exists yJohn)Finds(y, [\exists xUnicorn]),$$

which means that both sentences are analyzed as having the same logical form. In both cases we have interpreted the quantifier phrase 'a unicorn' as being nominalized, in which case we say that the referential concept it stands for has been deactivated, i.e., we are not here actively referring to a unicorn.

Now although these two sentences have the same logical form, there is a difference in the content of the predicate in each. In particular, whereas the predicate *Find* is extensional, the predicate *Seek* is not. Thus, given the extensionality of *Find*, we have as a meaning postulate:

$$[\lambda yFinds(y, [\exists xA])] = [\lambda y(\exists xA)Finds(y, x)],$$

where A is a name variable. What the meaning postulate stipulates here is that 'to be a y such that y finds an A' is 'to be a y such that some A is such that y finds it'. From this meaning postulate it follows that there is a unicorn that John finds.[26] The same cannot be said for *Seek*, on the other hand, because *Seek* is an intensional and not an extensional verb. In this way we are able to give the same sort analysis that Montague gave in his treatment of quantifier phrases in English, but without having to resort to Montague's sense-denotation type theory.

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25 The generalized schema for a "nominalized" quantifier phrase is:

$$[QxA] =_{df} [\lambda y(\exists G)(y = G \wedge (QxA)G(x))],$$

where A is a name variable and Q is a quantifier. Note that when nominalizing a quantifier phrase we replace the parentheses that normally occur as part of the phrase with brackets.

26 This meaning postulate is of course an instance of the more schematic form:

$$[\lambda yFinds(y, [QxA])] = [\lambda y(QxA)Finds(y, x)],$$

which can be applied to 'finds a few books', 'finds most books', etc.

5. Nominalization with Formula Operators

Now there is a problem with both Montague's and our analysis of a (complex) predication based on a relation with a quantifier phrase as direct object. Consider, e.g., the sentence 'Jim bought and ate an apple', the truth conditions of which can be symbolized as:

$$(\exists x Jim)(\exists y Apple)(Bought(x, y) \wedge Ate(x, y)).$$

As a speech or mental act, however, this won't do because of the double quantifier phrase. That is, as a speech or mental act what we have on this analysis are two active referential concepts, whereas in fact there is only one active reference in the assertion, namely to Jim.

We could try the following analysis in which the quantifier phrase 'an apple' has been nominalized and therefore deactivated:

$$(\exists x Jim)[\lambda x[\lambda y(Bought(x, y) \wedge Ate(x, y))]([\exists y Apple])](x).$$

This analysis will not do as well, however, because by λ-conversion and the extensionality of *Bought* and *Ate*, it is equivalent to 'Jim bought an apple and Jim ate an apple', which is not equivalent to the original sentence, because on this analysis the apple he bought might be different from the apple he ate.

I have made several proposals elsewhere as to how to answer this problem in our conceptualist logic.[27] We will not review those proposals here, but I want to suggest a new, different proposal that is based on an issue regarding how to interpret tense and modal operators – and probably formula operators in general – in our speech or mental acts. I think this proposal works best for the example we are considering here, at least for our conceptualist logic even if not also for Montague's sense-denotation type-theoretical view.

Consider, for example, the denial that the round square is round, which, using the negation sign for the formula operator 'it is not the case', we can symbolize as

$$\neg(\exists_1 x Square/Round(x))Round(x).$$

Here, of course, we are not referring to a square that is round. Nor of course are we predicating of the round square that it is not round (as Alexis Meinong would have it). Rather we are *denying* or rejecting the proposition that the round square is round. One way to read this is as 'That the round square is round *is not the case*', i.e., to understand the operator as a predicate, so that the above can also be symbolized as:

27 See Cocchiarella (2007, chapter 7.8).

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$Not([\exists_1 xSquare/Round(x))Round(x)])$,

where the bracketed formula is read as the nominalized form of the indicated sentence and *Not* is taken as a predicate. Here we represent the nominalization of a formula by bracketing it. Predication, needless to say, is not activated in a nominalized sentence. Nor of course is reference, i.e., there is no active reference to a round square in this denial.[28] What a nominalized sentence denotes in our conceptualist logic is the intension of the sentence.

Now the point to our proposal is that the same kind of analysis applies to tense and modal operators, and in particular to the operator $\mathcal{P}$ for 'it was the case', so that 'It was the case that φ', where φ is a formula can be symbolized as $\mathcal{P}\varphi$, but understood as $Was([\varphi])$, i.e., as 'that φ *was the case*'. Thus, when we assert the sentence 'That there is a king of France was the case', we are not actively referring to a king of France; rather we are only asserting that the proposition in question was the case or true in the past.

Why bring in the past-tense operator here when tense operators and their logic have been left as implicit in the background logic? The answer is because the predicate 'bought and ate' is in the past tense and really contains an implicit tense operator such as in 'x bought an apple *and then* ate it', which can be symbolized as:

$\mathcal{P}(\exists yApple)(\mathcal{P}Buys(x, y) \wedge Eats(x, y))$,

where $\mathcal{P}(\exists yA)(\mathcal{P}\varphi \wedge \psi)$ can be read as 'That $(\exists yA)(\varphi$ *and then* $\psi)$ *was the case*'. The 'and-then', in other words, can be defined in terms of the past-tense operator as indicated.[29] Notice that because the apple was eaten and hence no longer exists, we are not here (now) referring to an apple. The formula following the tense operator is to be interpreted as deactivated, in other words. The original sentence 'Jim bought and ate an apple' can now be symbolized as:

$(\exists xJim)[\lambda x\, \mathcal{P}(\exists yApple)(\mathcal{P}Buys(x, y) \wedge Eats(x, y))](x)$,

and although the quantifier phrase 'an apple' has not been nominalized in this formula, nevertheless, because of the past-tense context, the referential concept it stands for has been deactivated.

28 If we were to use this notation, we would need to add a meaning postulate connecting it to the standard notation so that we can engage in the usual deductions in the standard way. For convenience, we choose not to bother with introducing the new notation.

29 See Prior (1967:182). Prior did not use the 'and-then' operator for the purpose we are proposing here, however.

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6. The Identity (Two-Name) Theory of the Copula

The idea that the direct (or indirect) object of a relational predicate is deactivated in a speech or mental act applies no less so to the copula in its use as a relational predicate than it does to transitive verbs such as 'seek' and 'find'. In addition, this application to the copula is quite useful in our analysis of predication.

Now the medieval terminist logicians and linguists interpreted the copula as an identity, a view that most modern philosophers and linguists reject, but that is because the terminist logicians took the copula to be no less than predication itself (for categorical sentences). On our view, however, taking the copula as predication is no more correct than taking any relation (such as 'seek' or 'find') as a form of predication, which does not mean that it cannot be the relational basis of a complex monadic predication.

We will use the expression '*Is*' to represent the copula as a two-place predicate, and we note that, like the transitive verb 'find', the copula *Is* is extensional. Our meaning postulate in this regard transforms the copula into a strict identity. With a nominalized quantifier phrase as the direct object of the copula, the schematic meaning postulate for *Is* is as follows:

$$[\lambda x Is(x, [\exists y A])] = [\lambda x(\exists y A)(x = y)],$$

where A is a name variable. Thus, an assertion of the sentence 'Jan is a teacher' can now be symbolized as:

$$(\exists x Jan)Is(x, [\exists y Teacher]),$$

which, by our meaning postulate regarding the extensionality of the copula, is equivalent to:

$$(\exists x Jan)(\exists y Teacher)(x = y),$$

which is a more perspicuous representation of its truth conditions. It is important to keep in mind here that the predicate in the speech act that Jan is a teacher is $[\lambda x Is(x, [\exists y Teacher])]$, and not the copula or the identity sign itself. In other words, it is the predicable concept that this predicate stands for that is exercised in the speech act in question (and mutually saturated by the referential concept regarding Jan), and not the relational (identity) concept that *Is* stands for. As already noted, we have avoided writing out the λ-abstract throughout only for convenience.

It was something like the above sort of analysis that came to be called the two-name, or identity, theory of the copula in terminist logic. Apparently, Ockham and other terminists thought that every affirmative categorical proposition amounted

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to asserting an identity between the personal suppositions of the subject and predicate terms of the proposition, as, e.g., the suppositions of the names 'Jan' and 'teacher' in an assertion of 'Jan is a teacher'. A negative judgment was construed as a denial of such an identity. As a result, the identity theory of the copula came to be developed as a theory of the truth conditions of categorical propositions, a theory that is now referred to as *the doctrine of supposition proper*.[30]

Consider, for example, an assertion of 'Every whale is a mammal'. On our analysis this sentence has the logical form,

$(\forall x Whale)Is(x, [\exists y Mammal])$,

which, by the meaning postulate for *Is*, reduces to:

$(\forall x Whale)(\exists y Mammal)(x = y)$.

What this means in terminist logic is that *each* supposition of the categorical term 'whale' and *some* supposition of the categorical term 'mammal' are identical.

The terminists also understood categorical propositions with a predicate adjective as being based on the 'is' of identity as well. Thus although an assertion of 'Every raven is black' is represented in our logic by

$(\forall x Raven)Black(x)$,

this does not mean (as some philosophers have thought) that the terminists interpreted the sentence as an identity between ravens and black, or blackness, or separate blacknesses for different ravens. Rather, the predicate adjective 'black' was interpreted as an attributive adjective, so that to say a thing is black is to say that it is a black thing.[31] Predicate adjectives, in other words, were analyzed by the terminists as attributive adjectives applied to the common name 'thing', which is the opposite of our view that attributive adjectives are to be analyzed as predicate adjectives in relative clauses. But, as already noted, the common name 'black thing' is equivalent to the complex common name 'thing that is black', which is symbolized in our system as '$Thing/Black(x)$'. Thus, whereas the terminist logician would interpret 'Every raven is black' as 'Every raven is a black thing', we can represent the terminists' analysis as 'Every raven is a thing that is black' as follows:

$(\forall x Raven)Is(x, [\exists y Thing/Black(y)])$,

which, by our meaning postulate for *Is*, is equivalent to

30 See Scott (1966a: 30).

31 See Normore (1985: 194).

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$(\forall x Raven)(\exists y Thing/Black(y))(x = y)$,

which, by our meaning postulates for the relative clause operator, reduces in logic to the above formula with 'black' as a predicate adjective. What this shows is that we have not really escaped the predication of 'is black' by 'thing that is black', since the occurrence of 'is black' now occurs in the relative clause, and a repeat of the analysis would only go on in an infinite regress to 'thing that is a thing that is … a thing that is …'. In the end we must ground the predication in terms of the mutual saturation of a referential concept such as is expressed by 'Every raven' and a predicable concept such as is expressed by 'is black'.

Negative categorical sentences such as 'No whale is a fish' were interpreted by the terminists as denials or negations. That is, to assert that no whale is a fish is to deny that some whale is a fish:

$\neg(\exists x Whale) Is(x, [\exists y Fish])$,

which, by the meaning postulate for *Is* reduces to

$\neg(\exists x Whale)(\exists y Fish) Is(x, y)$,

and which for the terminists amounted to a denial that some supposition of the name 'whale' is identical with a supposition of the name 'fish'.

Finally, let us turn to the logical form of a negative particular categorical sentence such as 'Some raven is not black'. This sentence can be symbolized with a λ-abstract as follows:

$(\exists x Raven)[\lambda x \neg Black(x)](x)$.

Our use of the λ-abstract here is so as to emphasize that the negation is an internal part of the predicate. This sentence would be understood in terminist logic as the statement that 'some raven is not a black thing', which is symbolized as

$(\exists x Raven)[\lambda x \neg Is(x, [\exists y Thing/Black(x)])](x)$.

This last formula, by λ-conversion and the meaning postulate for *Is*, is equivalent to

$(\exists x Raven)(\forall y Thing/Black(x))(x \neq y)$,

the truth conditions for which are that some supposition of the common name 'raven' is not identical with any supposition of the complex common name 'thing that is black'.

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We will not go on with further examples of the terminists' two-name or identity theory of the copula.[32] It is important to note here, however, that the idea of the copula as an identity is not wrong, but it needs to be seen as restricted to contexts with a quantifier phrase occurring as the direct object of the copula, and even then, it is to be understood as the basis of a complex relational predicate. This applies even when the copula is between two proper names, as in 'Cicero is Tully', which can symbolized as:

$$(\exists x Cicero)[\lambda x Is(x, [\exists y Tully])](x),$$

but which – once names can be transformed into terms (in the extended logic that is to follow) – is equivalent to:

$$Cicero = Tully.$$

The important thing is to distinguish here the difference between the logical form that represents the speech act as based on a referential and predicable concept and the logical form that perspicuously represents its deductive role and hence its truth conditions (which in this case is the deductive logic of identity).

7. Plural Reference and Predication

We now turn to an analysis of plural reference and predication. As we will see, our account of the unity of predication in a speech or mental act applies no less so to plural reference and predication – and also to mass noun reference and predication – than to singular reference and predication. Both plural and singular predication, we maintain, are categorially on a par, i.e., there is no category difference or difference of level in English between singular and plural predication the way there is between subjects and predicates, though that is not how some philosophers represent the situation.[33] Plural reference will of course be to pluralities, and pluralities will be what plural predication is about. What we need accordingly are logical forms that represent pluralities, and along with such forms a logic of the truth conditions of our references to, and predications of, pluralities.

Now it should be noted that our treatment of names, proper or common, is not restricted to count nouns. That is so because the logic we have developed for count nouns applies to mass nouns as well. One important move in developing a logic for both plurals and mass nouns is to allow the common names that have so

32 See Cocchiarella (2001) for more examples.

33 Boolos (1986), e.g., takes second-order predicate logic to be the logic of plurals, and others such as MacKay (2006) take it to be a totally new syntactic category.

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far occurred exclusively as parts of quantifier phrases to occur also as "terms" or arguments of predicates, i.e., as denoting expressions on a par with the variables of first-order logic. That of course is exactly how proper names in particular are normally represented in standard logic; but our point is that the same is to be done with common names as well (which is essentially the way they are understood in Leśniewski's logic of names). And when such a common name is a count noun and occurs as an argument of a predicate then it is understood to denote a plurality (unless it denotes nothing).

In other words, we will allow for the nominalization or transformation of all names, proper or common, into "terms", i.e., expressions that can occur as arguments of predicates (the way the variables of first-order logic do).[34] But for now we will restrict our attention to count nouns. When a count noun such as 'man' is nominalized in the manner indicated, what we get in English is the plural 'men' or the noun 'mankind' as when we say that Socrates is a member of mankind, not meaning that Socrates is a member of the set of people, but only that he is one among men (or people). Being one *among*, or being a *member of*, a collective or group such as mankind does not mean being a member of a set. Membership has a meaning or use in contexts about pluralities as collectives other than sets, and it is that use that we are concerned with here. In addition to "nominalized" names, proper or common, conjunctions of names, as in 'George, Harry and Jim', are also taken to denote pluralities, or collective groups, as for example in the statement that Russell and Whitehead are the coauthors of *Principia Mathematica*, or that George, Harry, and Jim are playing cards (together).

Now because names, proper or common, and complex or simple, can be transformed into arguments of predicates, i.e., into terms, we need to add to our logic a variable-binding operator that generates complex names the way that the λ-operator generates complex predicates. We will use the cap-notation with brackets, $[\hat{x}A/\ldots x\ldots]$, for this purpose. Accordingly, where A is a name, proper or common and complex or simple, we take $[\hat{x}A]$ to be a complex name, but one in which the variable x is bound. Thus where A is a name and φ is a formula, $[\hat{x}A]$, $[\hat{x}A/\varphi]$, and $[\hat{x}/\varphi]$ are names in which all of the free occurrences of x are bound. We read these expressions as:

$[\hat{x}A]$ is read as '(the) A things',

34 To be a term in this logic is not the same as to be a singular term, i.e., a term that denotes at most one single object or thing. Pluralities in particular will be denoted by terms and so will the various parts of what mass nouns denote. By a term we mean only an expression that can occur as an argument of predicates.

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$[\hat{x}A/\varphi]$ is read as '(the) A things that are φ', and
$[\hat{x}/\varphi]$ is read as '(the) things that are φ'.

Thus for example, the conjunction of 'George, Harry, and Jim' can be represented by $[\hat{x}/x = George \vee x = Harry \vee x = Jim]$, which is read 'the things that are either George, Harry, or Jim'. Usually, though, it is just read as a conjunction of names, as in 'George, Harry and Jim are playing poker'.

Now we assume that in a given speech context we can distinguish the common names that represent count nouns from those that represent mass nouns. We do not assume, on the other hand, that there is an absolute, fixed distinction between count and mass nouns, but only that we can make such a distinction in particular contexts of use of language.[35] Plural reference in English, of course, involves only count nouns.

In regard to the logical forms for plural reference and predication, we extend the inductive definition of the meaningful (well-formed) expressions of our conceptualist logic to include the following clauses:

(1) if A represents a common count noun (in a given context), then A^P is a *plural name* (in that context);
(2) if A represents a common count noun (in a given context), x is a first-order variable, and φx is a formula, then $[\hat{x}A/\varphi x]^P$ and $[\hat{x}/\varphi x]^P$ are *plural names* (in that context);
(3) if $A/\varphi(x)$ represents a (complex) common count noun (in a given context), then $(A/\varphi x)^P$ is $A^P/[\lambda x \varphi x]^P(x)$ and $[\hat{x}A/\varphi x]^P$ is $[\hat{x}A\,^P/[\lambda x\varphi x]^P(x)]$;
(4) if F is a one-place predicate constant, or of the form $[\lambda x\varphi(x)]$, then F^P is a one-place *plural predicate constant*; and
(5) if A^P is a plural name (in a given context), x is a first-order variable, and φ is a formula, then $(\forall xA^P)\varphi$ and $(\exists xA^P)\varphi$ are formulas.

We read '$(\forall xMan^P)$' as the plural phrase 'all men' and '$(\exists xMan^P)$' as the plural phrase 'some men'. In other words we take these phrases as representing plural referential concepts. Similarly, we read the complex predicate $[\lambda x(\exists yMan)(x = y)]^P$ as the plural predicate 'men'. We note that a *plural* name is not a name *simpliciter*, and hence a plural name does not occur as a term or argument of predicates. That role is already filled by nominalized names *simpliciter*. That is the main difference here between plural names and names *simpliciter:* plural names do not occur as terms in our basic logical forms, which means that plural names occur only in

35 See, e.g., Pelletier (1975: 456), regarding the notion of a "universal grinder" that can change a count noun into a mass noun in a given context.

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the logical forms that represent our speech or mental acts, and in particular as plural predicates in plural predication or as part of plural quantifier phrases in plural reference. Finally, it is important to note that we use meaning postulates to connect the logical forms of our speech or mental acts with the logical forms that give a more perspicuous representation of the deductive implications of our speech or mental acts, and hence of their truth conditions.

We note that only monadic predicates are pluralized. A two-place relation R can be pluralized in either its first- or second-argument position, or even in both, by using a λ-abstract, as, e.g.,

$[\lambda x R(x,y)]^P$,
$[\lambda y R(x,y)]^P$,
$[\lambda x[\lambda y[R(x,y)]^P(y)](x)]^P$,

respectively; and a similar observation applies to n-place predicates for $n > 2$.

In turning to examples of plural reference and predication, let us consider the sentence 'Whales are mammals', where of course by 'whales' we mean 'all whales'. As a thought or speech act, the sentence can be symbolized as follows:

$(\forall x Whale^P) Mammal^P(x)$.

This formula of course should be equivalent to 'Every whale is a mammal', a statement that involves only singular predication. In fact, the equivalence is provable in the logic we describe below. It is based on the same logical considerations given for 'All men are mortal' being equivalent to 'Every man is mortal' described below.

For an example that is not reducible to singular predication, consider the statement that some men are playing poker (together), which we can symbolize as follows:

$(\exists x Man^P) Playing\text{-}poker^P(x)$.

Playing-poker is interpreted here as plural because the men are playing poker together. To be sure, if some men *are* playing poker (together), then some man, e.g., George, *is* playing poker, where 'playing poker' is now in the singular. But playing poker together, i.e., where 'playing poker' is pluralized, is not a distributive predicate, which means that it cannot be reduced to, or completely analyzed in terms of, 'playing poker' in the singular.

The question now is how are we to understand the truth conditions of a statement of the above form, or more generally of the form $(\exists x A^P)F^P(x)$, and also of the form $(\forall x A^P)F^P(x)$ as well? It is one thing to say that pluralities are the values of the variables in a formula, but what are pluralities, and how are they to be represented

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here both as to what we are referring to and what the predicate says about them? What, in other words, is the semantics or logic of pluralities?

Our answer is that a plurality is essentially what Bertrand Russell called a class as many as a plurality in his 1903 *Principles of Mathematics*, by which Russell did not mean either a set or a class as an abstract entity.[36] To be sure, Russell gave up the notion of a class as many as a plurality after 1903 as hopelessly confused, and that is just what Peter Geach claimed when he described it as "radically incoherent".[37] But Geach was wrong, because by following Russell s three basic principles regarding classes as many we have been able to formalize a logic of classes as many that is not only coherent but consistent as well.[38]

Russell assumed three important features about his notion of a plurality as a class as many, and, as indicated, each is valid in our logic of classes as many. The first is that a vacuous common count noun, i.e., a common name that names nothing, denotes nothing, which is not the same as having an empty class as its extension. Thus, according to Russell, "there is no such thing as the null class [in the sense of a class as many as a plurality], though there are null class-concepts", i.e., common-name concepts that have no extension, or denote nothing.[39] Thus, for example, the plurality, or class as many, of things that are not self-identical, $[\hat{x}/x \neq x]$, denotes nothing, i.e., $\neg(\exists y)(y = [\hat{x}/x \neq x])$ is a valid (provable) thesis of our logic of plurals, which it will be remembered is based on a free first-order logic. We use Λ as the symbol for the empty class as many, i.e.,

$$\Lambda =_{df} [\hat{x}/x \neq x].$$

As indicated, it is provable in our logic of classes as many as pluralities that there is no empty class as many, i.e.,

$$\neg(\exists x)(x = \Lambda)$$

is provable in our logic.

Now where membership in a plurality is defined as follows:

$$x \in y \leftrightarrow (\exists A)[y = A \wedge (\exists z A)(x = z)],$$

the Russell plurality, or class as many of things that are not a member of themselves, also does not exist, i.e.,

36 For a 17th century anticipation by Thomas Vincentius Tosca of the distinction between a class as many and a class as one see Angelelli (1979).

37 Geach (1980a: 225).

38 See, e.g., Cocchiarella (2002).

39 Cocchiarella (2002, §69).

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$$\neg(\exists x)(x = [\hat{y}/(\exists A)(y = A \wedge (y \notin A))])$$

is also a provable thesis of logic of plurals. Note, incidentally, that in the definition of $\in$ the occurrence of A in $(y = A)$ is as a term denoting the plurality of things that fall under the name concept that A stands for, whereas the occurrence of A in the quantifier phrase $(\exists zA)$ stands for the name concept itself.

The second important feature assumed by Russell is that the class as many as a plurality that is the extension of a count noun that names just one thing is just that one thing. In other words, unlike the singleton sets of set theory, which are not identical with their single member, the class as many that is the extension of a common count noun that names just one thing is none other than that one thing. This is not really odd in fact because we sometimes refer to, or speak of, e.g., 'some people' in a context when, it turns out, there is just one person involved, just as we also occasionally use 'some person' in a context when there is more than one person involved. In addition, overall the logic of pluralities comes out much more smoothly when we allow that a plurality might consist of just one thing. Thus given that 'Socrates' denotes one thing, then

$$(Socrates \in Socrates) \wedge (\forall x/x \in Socrates)(x = Socrates)$$

is a true sentence, and similarly so is

$$(\exists xSocrates)(\forall ySocrates)(x = y).$$

Finally, the third feature cited by Russell is that, unlike sets, classes as many are literally made up of their members, which is why they are also called pluralities (*Vielheiten)*, and not things that can themselves be members of classes.[40] Thus, according to Russell, "though terms may be said to belong to … [a] class [as many], the class [as a plurality] must not be treated as itself a single logical subject".[41] Thus, if a plurality or class as many is made up of at least two objects, then that plurality cannot itself be a member of a class as many, i.e., it cannot be one among a plurality.

In turning to the semantics or logic of classes as many as pluralities, we note that inclusion in a class as many, or being part of a plurality, can be defined in terms of membership, i.e., the relation of being among:

40 The idea that sets also have their being in their members is philosophically problematic when we consider the empty set, which would mean that it has its being in nothing, and hence that it itself is nothing. But the empty set is essential to pure set theory and therefore is not nothing.

41 Russell (1903, §70).

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$$x \subseteq y \leftrightarrow (\forall z)[z \in x \rightarrow z \in y].$$

As indicated, the inclusion relation, $\subseteq$, of our logic of classes as many can also be used to represent the part-to-whole relation that is essential to any account of mass-noun reference and predication. The logic of mereology as described in the Leonard-Goodman's calculus of individuals is in fact reducible to our logic of classes as many, or at least the atomistic (free-logic) version is.[42] Proper inclusion, $x \subset y$, is definable of course in terms of inclusion:

$$x \subset y \leftrightarrow x \subseteq y \wedge y \nsubseteq x.$$

What is an "atom" (in the sense of our logic), or "single thing", i.e., an "individual", is analyzable in the logic of classes as many as follows:

$$Atom =_{df} [\hat{x}/\neg(\exists y)(y \subset x)].$$

That is, to be an atom, or individual, is to be something that has no proper subpart.[43] In the case of mass nouns, which we will deal with next, an atom is the same as a *minimal* part. There are a number of interesting features of the logic of classes as many as pluralities that we will not go into here, but the details of which can be found in my paper of 2002. We should also note, however, that combining pluralities of the same kind results in a plurality of that same kind:

$$(\forall x/x \subseteq A)(\forall y/y \subseteq A)[(x \cup y) \subseteq A].$$

In other words, pluralities are cumulative. As we will see, so too are what mass nouns denote.

Now the semantics or logic underlying plural reference is given in terms of the logic of classes as many as pluralities.[44] As already noted, we use meaning postulates to connect the logical forms at the level of our speech and mental acts with the logical forms at the level of the underlying logic of classes as many. Thus, for the plural reference of 'Some A^P are …', we have the following principle as a meaning postulate (for plurals):

$$\text{(MPP1)} \qquad (\exists x A^P)\varphi(x) \leftrightarrow (\exists x/x \subseteq [\hat{y}A])\varphi(x).$$

42 See Eberle (1970, Chapter 2), for a reconstruction of the calculus of individuals in a free first-order logic.

43 This 'atom' terminology goes back to Nelson Goodman and the so-called Leonard-Goodman calculus of individuals.

44 For a set-theoretic semantics as well see the appendix of Cocchiarella (2002).

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Accordingly, for the statement that some men are playing poker we have the following biconditional connecting the logical form of the speech act with the logical form of the truth conditions of that act:

$$(\exists x Man^P)Playing\text{-}poker^P(x) \leftrightarrow (\exists x/x \subseteq [\hat{y}Man])Playing\text{-}poker^P(x).$$

Now given that 'playing poker' is a distributive predicate in only one direction, i.e., that

$$Playing\text{-}poker^P(x) \rightarrow (\forall y/y \in x)Playing\text{-}poker(y)$$

is true, then the right-hand side of the above biconditional is provably equivalent to the formula

$$[\hat{y}\ Man/Playing\text{-}poker(y)] \neq \Lambda.$$

That the plurality (class as many) of men playing poker is not empty is equivalent of course to saying that some men are playing poker.

Let us turn now to how the universal plural 'All^P', the meaning postulate of which is given as follows:

$$\text{(MPP2)} \qquad (\forall x A^P)\varphi(x) \leftrightarrow (\forall x/x \subseteq [\hat{y}A])\varphi(x).$$

Now given that the cognitive structure of an assertion of 'All men are mortal' is represented as,

$$(\forall x Man^P)Mortal^P(x),$$

then, by (MPP2), it follows that, semantically, the assertion amounts to predicating mortality to every plurality of men,

$$(\forall x/x \subseteq [\hat{y}Man])Mortal^P(x),$$

which is equivalent to saying that the members of the entire group of men taken collectively *are* mortal:

$$(\forall x/x = [\hat{y}Man])Mortal^P(x).$$

In conceptualist terms, this formula comes very close to what Russell claimed in his 1903 *Principles*, namely, (to use Russell's terminology) that the denoting phrase 'All men' in the sentence 'All men are mortal' denotes the class as many of men.

Note that the predicate 'mortal' is distributive (in both directions), a fact that is represented by the following meaning postulate:

$$(\forall x)[Mortal^P(x) \leftrightarrow (\forall y/y \in x)Mortal(y)].$$

It follows, accordingly, that the statement that all men *are* mortal,

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$(\forall x Man^P) Mortal^P(x)$,

is provably equivalent to the different statement that every man *is* mortal:

$(\forall x Man) Mortal(x)$.

Of course, there are plural predicates that are not distributive (in either direction) and which therefore cannot be reduced in this way.

Plural identity, as in 'The triangles that have equal sides *are* (identical with) the triangles that have equal angles', is based on the plural of the copula for both subject and direct object. The relation *Are* as the plural of *Is* can be represented by the following λ-abstract:

$Are =_{df} [\lambda x[\lambda y Is(x,y)]^P]^P$.

For convenience, we will ignore going through several applications of the meaning postulate about *Is* that leads to a reduction of *Are* to an identity between terms. We will simply use the identity sign instead. In other words, although 'A^P *are* B^P' is definable as $[\lambda x[\lambda y Is(x,B^P)]^P]^P(A^P)$, it is simpler for our present purpose here to represent 'A^P *are* B^P' simply as '$A = B$', with which, by meaning postulates, $[\lambda x[\lambda y Is(x,B^P)]^P]^P(A^P)$ is provably equivalent.

Plural identity, we note, is involved in the truth conditions for statements with two plural definite descriptions, just as singular identity is involved in the truth conditions for two singular definite descriptions. Thus, just as the statement that the spy is the bald man can be symbolized as follows:

$(\exists_1 x Spy) Is(x, [\exists_1 y Man/Bald(y)])$,

which reduces to:

$(\exists_1 x Spy)(\exists_1 y Man/Bald(y))(x = y)$,

so too in an entirely similar way we can formalize the statement that the spies are the bald men:

$(\exists_1 x Spy^P) Are(x, [\exists_1 y Man^P/Bald^P(y)])$,

which reduces to:

$(\exists_1 x Spy^P)(\exists_1 y Man^P/Bald^P(y))(x = y)$.

Now the logical analysis of a statement of the form The A^P *are* F^P can be given as follows:

$(\exists_1 x A^P) F^P(x) \leftrightarrow (\exists x A^P)[(\forall y A^P)(y = x) \wedge F^P(x)]$,

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for when the plural definite description is being used with existential presupposition. This of course is similar to the Russellian analysis for the singular form 'The *A* is *F*'. From this it follows that if *F* is distributive then the plural definite description 'The A^P *are* F^P' denotes the class as many of *A* that are *F*, i.e.,

$$(\exists_1 xA^P/F^P(x))(x = [\hat{y}A/F(y)]),$$

which explains why we read $[\hat{y}A/F(y)]$ as the 'A's that are *F*'.

For a sentence of the form 'The A's that are *F* are the B's that are *G*', as in 'The triangles that have equal sides *are* (identical with) the triangles that have equal angles', the analysis or symbolization can be given as follows:

$$(\exists_1 xA^P F^P(x))Are(x, [\exists_1 yB^P/G^P(y)]),$$

which in our logic reduces to:

$$(\exists_1 xA^P F^P(x))(\exists_1 yB^P/G^P(y)])(x = y),$$

which of course can be further reduced by means of the above formula for plural definite descriptions.

Finally, we note that plurals also can be used with numerical quantifier phrases, as in 'There are twelve Apostles', as well as with numerical predicates, as in 'The Apostles *are* twelve'. As is well-known, the predicate 'twelve' can be defined in terms of numerical quantifier phrase, such as $(\exists^{12}y)$. For example, one analysis of 'The apostles are twelve' is the following:

$$(\exists_1 xApostle^P)[\lambda x(\exists^{12}y)(y \in x)]^P(x).$$

The quantifier phrase $(\exists^{12}y)$ of course is readily definable in first-order logic (with identity).[45] We can also define the predicate 'twelve' more generally as the concept of pluralities that have twelve members:

$$12 =_{df} [\lambda x(\exists A)(x = A \wedge (\exists^{12}yA)(y \in x))].$$

A similar analysis applies of course for each natural number. 'The Apostles are twelve' can now be simply represented as:

$$(\exists_1 xApostle^P)12(x).$$

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45 We note that given the extensionality of the membership predicate $\in$ (and its converse) the predicate $[\lambda x(\exists^{12}y)(y \in x)]$ is provably equivalent to $[\lambda x \breve{\in}(x, [\exists^{12}y])]]$, where $\breve{\in}$ is the converse of $\in$. The active quantifier $(\exists^{12}$y) in the predicate of the above formula can then be replaced by one that is deactivated for the speech act in question. We avoid doing so here for convenience.

The sentence 'There are twelve Apostles' is more involved, however, because of the presence of two seemingly active quantifier phrases, namely 'twelve' and 'There are'. An alternative reading of this sentence is 'Twelve Apostles there are', where the quantifier phrase 'there are' is now (part of) the predicate. Treating 'there are' as (part of) the predicate amounts to pluralizing 'there is' as a predicate, specifically as $[\lambda x(\exists y)(x = y)]^P$.[46] The sentence 'Twelve Apostles there are' can then be analyzed as:

$$(\exists^{12} x Apostle^P)[\lambda x(\exists y)(x = y)]^P(x),$$

which is reducible to:

$$(\exists^{12} x Apostle)(x = x),$$

which amounts to saying indirectly that there are twelve Apostles.[47] A similar analysis can be given more generally for sentences of the form 'There are *n* many A^P'.

We conclude that our conceptualist account of predication in terms of the mutual saturation of referential and predicable concepts as unsaturated cognitive capacities applies to plural predication as well as to singular predication. In addition we have shown how sentences based on plural reference and predication can be analyzed in terms of the logically perspicuous forms of our conceptualist logic, which includes a logic of classes as many as pluralities.

8. Mass Noun Reference and Predication

We now turn to our analysis of mass-noun reference and predication. The details of our analysis are similar to that for plural reference and predication, which is not surprising given the grammatical similarity between plurals and mass nouns, such as the fact that mass nouns can take only those determiners that can also be used with plurals.[48]

Mass nouns can be complex, such as 'polluted water', 'tall grass', 'modern furniture', etc., as well as simple, such as 'milk', 'gold', 'furniture', etc. Some nouns in

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46 The predicate $[\lambda x(\exists y)(x = y)]^P$ can be replaced by $[\lambda x Is(x,[\exists y])]^P$, with which it is identical by meaning postulates, so that there is no active quantifier in the predicate.

47 Note that the consequent of the formula is equivalent to a trivially provable thesis, namely that whatever is in x is something:

$$[\lambda x(\exists y)(x = y)]^P(x) \leftrightarrow (\forall z/z \in x)(\exists y)(z = y),$$

which means that the consequent is equivalent to $x = x$.

48 For an account of the similarities between mass nouns and plurals, see, e.g., Nicolas (2008).

some contexts function as mass nouns and in other contexts as count nouns, e.g., 'chicken' as in 'George ate a lot of chicken' (mass noun) and 'George has five chickens in his back yard' (count noun).

As already noted for count nouns, we assume that we can distinguish in a given speech context the common names that represent mass nouns from those that represent count nouns; but we do not assume that there is an absolute, fixed distinction between count and mass nouns.[49]

In regard the logical forms for expressing mass-noun reference and predication, we extend the inductive definition of the meaningful (well-formed) expressions of our conceptualist framework to include the following clauses:

(1) if A represents a mass noun (in a given context), then A^M is a *mass name* (in that context);
(2) if A represents a mass noun (in a given context), x is a first-order variable, and φx is a formula, then $[\hat{x}A/\varphi x]^M$ and $[\hat{x}/\varphi x]^M$ are (complex) *mass names* (in that context);
(3) if $A/\varphi(x)$ represents a (complex) mass noun (in a given context), then $(A/\varphi x)^M$ is $A^M/[\lambda x\varphi x]^M(x)$ and $[\hat{x}A/\varphi x]^M$ is $[\hat{x}A/^M[\lambda x\varphi x]^M(x)]$; and
(4) if A^M is a mass name (in a given context), x is a first-order variable, and φ is a formula, then $(\forall xA^M)\varphi$ and $(xA^M)\varphi$ are formulas.

We note that clause (3) is needed to allow for the representation of such complex mass nouns as 'polluted water', i.e., 'water that is polluted'. In addition, we note that in referring to all, some, a lot of, etc., water, we are referring to parts of water that have indefinitely many subparts that are again parts of water and that stand in the same relation to the larger part as the larger part to the whole. But having indefinitely many subparts, we note, is not the same as having infinitely many subparts; that is, it does not mean that there is an infinite descent of subparts.[50] The various parts of water are not discrete, well-delineated single objects, i.e., "individuals", in the way that the minimal parts are, namely, the molecules of water, which are discrete and well-delineated. Thus, although the minimal parts of what a mass noun denotes are individuals, and hence can be individuated, it is only the non-minimal parts that are problematic in this way.

We note that as with plural names *mass* names are not names *simpliciter*, and hence a mass name cannot occur as a term or argument of predicates. That role

49 We will not deal with abstract mass nouns in this paper.

50 Having indefinitely many subparts is different from having infinitely many subparts partly because subparts themselves might have subparts, and in general there is no principle way to count all of the subparts.

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is already done by nominalized names *simpliciter*. That is the main difference here between *mass* names and names *simpliciter:* mass names do not occur as terms in our basic logical forms. In other words, mass names, like plural names, occur only in the logical forms that represent our speech or mental acts, and in particular as mass predicates in their role as mass predications or as part of mass quantifier phrases for mass reference. Finally, it is important to emphasize once again that we use meaning postulates to connect the logical forms of our speech or mental acts with the logical forms that give a more perspicuous representation of the deductive implications of our speech or mental acts, and hence of their truth conditions.

Now it is noteworthy that our underlying logic of classes as many contains an atomistic (free-logic) mereology as a subsystem, which of course is natural for pluralities – but not, according to some linguists and philosophers, for the semantics of mass nouns. Nevertheless, despite the negative view, we maintain that an atomistic mereology is appropriate for mass nouns. In our logic we can speak of the extension (at a given time) of a mass noun, which in fact consists just of the minimal parts of what that mass noun denotes (at that time). Of course that is not a problem for mass nouns representing elementary substances and chemical compounds, because such mass nouns in fact denote at a given time the total class as many of individual atoms or molecules of those substances and compounds existent at that time. The mass noun 'gold', for example, denotes now all of the gold, and therefore all of the gold atoms, that exists now; and 'water' similarly denotes now all of the water, and therefore all the molecules of water that exist now. Pieces or bits of gold and bodies of water, large or small, are all made up of the atoms of gold or molecules of water, and therefore they are really subpluralities – or subclasses of classes as many – of the total class as many, or plurality, of gold and water, respectively.

The mass nouns for space and time, to be sure, have traditionally been assumed not to have minimal parts, and of course mathematically that is a consistent assumption. In particular, it is not just conceivable, but consistent as well, that there should be an infinite descent of the parts of space and time. Nevertheless, it is noteworthy that in modern quantum physics space and time are "quantized", and therefore do have minimal parts. The "Planck length" of 10^{-33} cm. is the smallest length physically possible in quantum mechanics, and there is a smallest physically possible time as well, namely, the time it takes for light to cross the Planck length, which is 10^{-43} seconds. This means that real space and time are not infinitely divisible after all.

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Mass nouns other than those for elementary substances and molecules built up from such elements are also unproblematic, we maintain. The mass noun 'furniture', for example, clearly denotes all of the individual pieces of furniture, and similarly 'jewelry' and 'silverware' denote all the individual pieces of jewelry or silverware. Mass nouns such as 'wine', 'milk', 'coffee', etc., also clearly do not have an infinite, unending descent of parts that are also wine, milk or coffee.

Is there an atomistic theory for the semantics of mass nouns implicit in natural language or our commonsense framework? That is certainly not obvious; but it is an interesting hypothesis. It has been suggested by Henry Laycock, for example, who thinks that "a kind of atomic theory is implicit in the ordinary use of mass terms based on our experience of the behavior of stuff".[51] In other words, according to Laycock, we should "think of stuff as a plurality of things, each of the same kind for any given kind of stuff", and in that way we can "construe any mass term '*m*' as a plural sortal of the form '*m* elements'".[52] An atomistic mereology for mass nouns is appropriate, in other words, not just because there is no infinite descent in nature, but also because such a view would explain the source of the grammatical similarities between the semantics of mass nouns and that of plurals. A similar position is taken by Gennaro Chierchia who suggests that a "mass noun simple denotes a set of ordinary individuals plus all the pluralities of such individuals".[53] Chierchia recognizes that this "view is an 'atomistic' one: we are committed to claiming that for each mass noun there are minimal objects of that kind, just like for count nouns, even if the size of these minimal parts may be vague".[54]

In any case, as we will see, our atomistic logic of classes as many as pluralities works just as well for mass nouns as it does for plurals. In addition, one consequence of this logic is that each nominalized mass noun denotes at any given time the entire class as many of its minimal parts, i.e., the "atomic", minimal parts of the mass noun that exist at that time, which also means that it denotes the sum of all of its parts (existent at that time).

51 Laycock (1972: 39).

52 Laycock (1972: 38).

53 Chierchia (1998: 54). Unlike Chierchia, however, I assume that a mass noun occurring as a term denotes not the *set* of its minimal parts but, *qua* plurality, the class as many of its minimal parts. Also, whereas Chierchia maintains that every plurality consists of two or more members, I allow single individuals to be pluralities. (I did initially require groups to contain two objects in my 2002 paper, but I later changed that in my 2009 paper.)

54 Chierchia (1998: 54).

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The meaning postulates connecting the logical forms representing our speech and mental acts in the case of mass nouns are given in a way entirely analogous to those already stipulated for plural reference and predication. Thus, just as the plural count noun phrase 'some A^P', in symbols, $(\exists xA^P)$, was interpreted as $(\exists xA/x \subseteq A)$, which can be read as 'Some plurality of A things', so too a similar interpretation applies to mass nouns. That is, where A^M is taken as a mass name in a given context, then 'Some A^M' can also be read informally as 'Some part of A', which can then be interpreted as follows:

(Sm/Mass) $\quad (\exists xA^M)\varphi x \leftrightarrow (\exists x/x \subseteq A)\varphi x.$

Also, because 'All A^M' is the dual of 'Some A', then 'All A^M', which, when A^M is a mass name, can also be read informally as 'Every part of A', is similarly interpreted as:

(All/Mass) $\quad (\forall xA^M)\varphi x \leftrightarrow (\forall x/x \subseteq A)\varphi x.$

Thus a sentence such as 'Water is a fluid', by which of course we mean 'All water is a fluid' can be symbolized as follows, where the predicate 'is a fluid' is based on the copula:

$(\forall x Water^M) Is(x, [\exists y Fluid^M]),$

which by (All/Mass) and (Sm/Mass) and the identity theory of the copula reduces to:

$(\forall x/x \subseteq Water)(\exists y/y \subseteq Fluid)(x = y),$

which says that every part of water is part of some fluid.

Predication in the sentence 'Water is in the radiator' is not based on the copula but rather on the relation of being in the radiator, and of course it is not all water that is in the radiator, but only some water:

$(\exists x Water^M) In(x, [\exists_1 y Radiator]),$

which by the meaning postulate (Sm/Mass) and the extensionality (in the presumed context[55]) of the relation of being in the radiator reduces to

$(\exists x/x \subseteq Water)(\exists_1 y Radiator) In(x, y).$

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55 There are other uses of the relation "in" that are not extensional, such as the relation of "in" between a story or someone's belief space and the propositions that make up the content of that story or that person's belief space.

What these last two examples also show is that predication is not always interpreted in the same way when mass nouns are the grammatical subject; that is, it may be universal as in 'Water is a fluid' or particular as in 'Water is in the radiator'.

Another type of problem arises when more than one determiner or quantifier phrase occurs in a sentence. The sentence 'There is some water in the radiator' has two determiners, for example, namely 'There is' and 'some'. Most logic texts would interpret this as above, i.e., as 'Some water is in the radiator', which amounts to ignoring the first quantifier phrase 'There is'. An alternative reading is possible, however, and is similar to our earlier example with the plural sentence 'There are twelve Apostles', which we interpreted as 'Twelve Apostles there are'. The sentence 'There is some water in the radiator' would be interpreted then as 'Some water in the radiator there is', which would be symbolized as:

$$(\exists x Water^{M}/In(x, [\exists_1 y Radiator])[\lambda x(\exists z)(x = z)]^{M}(x),$$

which by the meaning postulate (Sm/Mass) and λ-conversion reduces to the above analysis for 'Some water is in the radiator', which of course is what it should reduce to.[56]

What about when a mass noun is not the subject but the predicate of a sentence? In predicate position, the phrase 'is water' is interpreted as 'x is some water', which, semantically, is interpreted as 'x is identical with some water', so that the 'is' in this case is really the copula. Thus, e.g., the sentence 'The puddle is water' is analyzed as:

$$(\exists_1 y Puddle)Is(y, [\exists x Water^{M}]),$$

which, by the meaning postulates for *Is* and (Sm/Mass), reduces to:

$$(\exists_1 y Puddle)(\exists x/x \subseteq Water)(y = x).$$

Finally, we note that because the logic of classes as many contains an atomistic mereology, we have as a theorem of the system that combining two parts of water results in another part of water, i.e.,

$$(\forall x/x \subseteq Water)(\forall y/y \subseteq Water)[(x \cup y) \subseteq Water]$$

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56 By the extensionality of $[\lambda x(\exists z)(x = z)]^{M}$, which is represented as follows:

$$(\forall x)([\lambda x(\exists z)(x = z)](x) \leftrightarrow (\forall w/w \subseteq x)(\exists z)(w = z)),$$

it follows that in the above context the predicate $[\lambda x(\exists z)(x = z)]^{M}(x)$ is vacuous and trivially equivalent to $x = x$. This predicate is provably identical with $[\lambda x Is(x, [\exists z])]^{M}$, in which the quantifier has been deactivated.

is provable in our logic, which of course parallels the same result for plurals. This is another respect in which mass nouns are like plurals. Another observation is that because being *part of* in our logic means being a subclass (as many) of, then it follows that mass nouns are provably distributive:

$$(\forall x A^M)(\forall y/y \subseteq x)(\exists z A^M)(y = z).$$

In other words, if x is some part of what a (nominalized) mass name A denotes, then every part of x is also some part of what A denotes. But let us note that even though a hydrogen atom and an oxygen molecule are in some appropriate sense "part of" some molecule of water, it does not follow that they also part of some water in our sense of *part-of*, namely $\subseteq$, which is based on membership in a class as many.

9. Concluding Remarks

Our basic thesis is that the unity of an assertion as a speech act, and of a judgment as a mental act, whether overtly expressed or not, consists in the mutual joint-exercise of an unsaturated referential and predicable concept in which each saturates the other in a kind of mental chemistry that results in an event, namely a mental act and a speech act as well if the mental act is overtly expressed. In our theory, referential and predicable concepts are unsaturated, complementary, intersubjective cognitive capacities that underlie our rule following abilities in the use of language, and in particular of our use of referential and predicable expressions. Referential expressions are represented in our logic by quantifier phrases restricted by a name, proper or common, and simple or complex. We have shown how our theory of predication applies to plural and mass noun reference and predication no less so than to predication in the indicative mode.

Our underlying logic for both plurals and mass nouns is a logic of classes as many as pluralities. The important new move in this logic is allowing a transformation of the names that are part of quantifier phrases into names that can occur as "terms", i.e., as arguments of predicates. What these new terms denote are pluralities, including the pluralities that consist of the minimal parts of what mass nouns denote. This logic has been developed in detail elsewhere, along with a set-theoretic semantics.[57] Both pluralities and the denotata of mass nouns (as, e.g., the various parts of water), are represented as being on the same logical level as individuals or single objects (each of which is itself a plurality or class as many of one), which is the way they are represented in natural language. This is

57 See Cocchiarella (2002).

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different from most other treatments today of plurals by philosophers of logic. But as I have shown elsewhere, the principal alternative approach to the logic of plurals is reducible to the approach described here.[58]

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58 See, e.g., Cocchiarella (2015). I want to thank Ignacio Angelelli and Woosuk Park for comments and corrections.

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